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On a possible estimation of the constituent-quark parameters from Jefferson Lab experiments on the pion form factor

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Abstract. The charge form factor of the pion is calculated for the momentum transfer range of the Jefferson Lab experiments. The approach is based on the instant form of the relativistic Hamiltonian dynamics. It is shown that the form-factor dependence on the choice of the model for the quark wave function in the pion is weak, while the dependence on the constituent-quark mass is rather significant. It is possible to estimate the mass of the constituent quark and the sum of the anomalous magnetic moments of the u- and $\bar{d}$-quarks from the JLab experiments.

At present, the constituent quark model (CQM) is widely and successfully used for the description of hadron properties at low and intermediate energies [1–14]. The reasons for this are well known: first, CQM uses the physically adequate degrees of freedom; second, CQM describes nonperturbative effects. These facts give the possibility to use CQM for the investigation of the so-called "soft" structure of hadrons, e.g., in exclusive processes, in contrast to QCD (see, e.g., [15]).

The main feature of CQM versus QCD is the extraction of a finite number of the most important degrees of freedom needed to describe the hadron. All dynamical effects of QCD are incorporated in CQM through the effective (constituent) quark mass and the internal quark structure in terms of the quark form factors. So in the framework of CQM, constituent quarks have all the material properties of free particles and interact with each other through the confinement potential. This means that the constituent quark is characterized by an effective mass, a mean-square radius (MSR) and an anomalous magnetic moment. Let us remark that the concept of extended constituent quarks also appears in some quantum field theory models, for example, in the Nambu–Jona–Lasinio model with spontaneous chiral symmetry breaking [16]. In this context one can imagine that CQM is initiated by QCD. However, it is very important for us to remind ourselves that CQM is not a direct consequence of QCD, but a very successful phenomenological model [17].

For the description of electroweak properties it is necessary to take into account the relativistic effects, which especially are large in systems of light quarks. We will use the relativistic Hamiltonian dynamics (RHD) [18], which is one of the approaches to describe the relativistic properties of CQM.

In the present paper we discuss the dependence of the electromagnetic pion form factor on the internal quark structure. The interest in this problem is due particularly to a possible interpretation of current experiments at Jefferson Lab of the measurement of the pion form factor [19] in the range of momentum transfer $0.5\,(\mathrm{GeV}/c)^2 < Q^2 < 5\,(\mathrm{GeV}/c)^2$. Using one of the relativistic forms of CQM we obtain the result that the pion form factor in this region of Q^2 depends strongly on the constituent-quark mass, while the dependence on the model of the quark interaction in the pion is weak. This fact gives hope that it could be possible to estimate the constituent-quark mass from Jefferson Lab experiments. With the use of the model independent Gerasimov sum rule [20], it is possible to estimate the anomalous magnetic moments of the constituent quarks from these experiments and our calculations. So, the important characteristics of CQM can be obtained.

It is necessary to make a remark on the applicability of the concept of the constituent-quark model in the case of the pion. As is well known (see, e.g., [21]) in the low energy region the pion can be considered as the Goldstone boson of spontaneously broken chiral symmetry in QCD with massless quarks. In such an approach one can describe the small value of the pion mass as well as the observed leptonic decay constant. However, in this approach the pion's internal structure, in particular the observed electromagnetic structure, cannot be described adequately (see, e.g., [22]).

On the other hand, the quark presence in the pion has been clearly demonstrated at high energy through muon pair production in Drell–Yan processes [23]. To use

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the pion quark concept at low energy one needs to take into account non-perturbative dynamical effects. In the frame of CQM the quark degrees of freedom appear as constituent quarks: the effective particles including the quark–gluon cloud. The constituent-quark structure usually is described in a phenomenological way by the set of parameters mentioned above, including the constituent mass. CQM cannot pretend, naturally, to describe the small value of the pion mass; however, it pretends to give a phenomenological description of the observed internal structure, e.g., its electromagnetic properties.

The concept of the pion as a Goldstone boson and that of the pion as a quark–antiquark composite system are in duality and at present have not yet been unified in the framework of one single approach, although attempts to this effect exist (see, e.g., [22]).

In this paper, we use the version of [24] of the instant form of RHD.

The charge form factor of the pion can be obtained from the electromagnetic current matrix element for a composite system in an arbitrary coordinate frame:

$$\langle p_\pi | j_\mu | p'_\pi \rangle = (p_\pi + p'_\pi)_\mu F_\pi(Q^2). \tag{1}$$

$F_\pi(Q^2)$ is the electromagnetic form factor of the pion, describing the transition dynamics; it is an invariant function. The 4-vector $(p_\pi - p'_\pi)_\mu$ describes the geometric (transformation) properties of the matrix element, p_π the 4-momentum of the pion.

In RHD the Hilbert space of composite particle states is the tensor product of single particle Hilbert spaces: $\mathcal{H}_{q\bar{q}} \equiv \mathcal{H}_q \otimes \mathcal{H}_{\bar{q}}$, and the state vector in RHD is a superposition of two-particle states. As a basis in $\mathcal{H}_{q\bar{q}}$ one can choose the following set of vectors:

$$\begin{aligned} |\boldsymbol{p}_1, m_1; \boldsymbol{p}_2, m_2\rangle &= |\boldsymbol{p}_1, m_1\rangle \otimes |\boldsymbol{p}_1, m_2\rangle, \\ \langle \boldsymbol{p}, m | \boldsymbol{p}', m'\rangle &= 2p_0 \delta(\boldsymbol{p} - \boldsymbol{p}')\delta_{mm'}. \end{aligned} \tag{2}$$

Here $\boldsymbol{p}_1, \boldsymbol{p}_2$ are the particle momenta and m_1, m_2 the spin projections.

Since we consider the two-quark system as one composite system, the natural basis is one with separated center-of-mass motion:

$$|\boldsymbol{P}, \sqrt{s}, J, l, S, m_J\rangle, \tag{3}$$

with $P_\mu = (p_1 + p_2)_\mu$, $P_\mu^2 = s$, $s^{1/2}$ is the invariant mass of the two-particle system, l the angular momentum in the center-of-mass frame, S the total spin, J the total angular momentum, m_J the projection of the total angular momentum.

The basis (3) is connected with (2) through the Clebsch–Gordan decomposition of the Poincaré group [24].

Now the decomposition of the electromagnetic current matrix element for the composite system (1) in the basis (3) has the form

$$\begin{aligned} &(p_\pi + p'_\pi)_\mu F_\pi(Q^2) \\ &\quad = \sum \int \frac{\mathrm{d}\boldsymbol{P}}{N_{CG}} \frac{\mathrm{d}\boldsymbol{P}'}{N'_{CG}} \mathrm{d}\sqrt{s}\mathrm{d}\sqrt{s'} \langle p_\pi | \boldsymbol{P}, \sqrt{s}, J, l, S, m_J\rangle \\ &\quad \times \langle \boldsymbol{P}, \sqrt{s}, J, l, S, m_J \mid j_\mu \mid \boldsymbol{P}', \sqrt{s'}, J', l', S', m_J{}'\rangle \\ &\quad \times \langle \boldsymbol{P}', \sqrt{s'}, J', {}', S', m_J{}' | p_\pi{}'\rangle. \end{aligned} \tag{4}$$

Here the sum is over the discrete variables of the basis (3). $\langle \boldsymbol{P}, \sqrt{s}, J, l, S, m_J | p_\pi\rangle$ is the composite system wave function,

$$\begin{aligned} &\langle \boldsymbol{P}', \sqrt{s'}, J', l', S', m'_J | p_\pi\rangle \\ &\quad = N_\pi \delta(\boldsymbol{P}' - \boldsymbol{p}_c)\delta_{JJ'}\delta_{m_J m'_J}\delta_{ll'}\delta_{SS'}\varphi^J_{lS}(k). \end{aligned} \tag{5}$$

$s = 4(k^2 + M^2)$, M is the quark mass; N_π and N_{CG} are factors due to normalization. The concrete form of N_π and N_{CG} will not be used.

The basis (3) is the relativistic analogy of the basis of the generalized spherical functions of non-relativistic quantum mechanics (see, e.g., [25]). In this basis the pion wave function is the eigenfunction of the operators $\hat{J}^2$, $\hat{J}_3$, $\hat{l}^2$ as well as of the operator of the total spin squared, $\hat{S}^2$, defined in the invariant way (see, e.g., [26]). All the operators have zero eigenvalues, because for the pion $J = l = S = 0$. The quark spin properties are taken into account in the basis (3) by the corresponding Clebsch–Gordan decomposition.

Using the fact that the right-hand side of (4) is covariant and considering (5) let us write (4) as follows:

$$\begin{aligned} &(p_\pi + p'_\pi)_\mu F_\pi(Q^2) \\ &\quad = \int \mathrm{d}\sqrt{s}\mathrm{d}\sqrt{s'}\varphi(k) A^{\mathrm{int}}_\mu(s, Q^2, s') g(s, Q^2, s')\varphi(k'). \end{aligned} \tag{6}$$

Here we use for simplicity the notation $\varphi^J_{lS}(k) \to \varphi(k)$, $g(s, Q^2, s')$ is the invariant part of the current matrix element which describes the transition dynamics, and A^{int}_μ is the covariant part of the matrix element which describes its transformation properties.

Equation (6) means that the two 4-vectors are equal and this equality is to be valid for any choice of the wave function $\varphi(s)$ of the two-particle system's internal motion. If the wave function is varied, then the scalar part of the l.h.s. (the form factor $F_\pi(Q^2)$) is changed, while the covariant part (the vector $(p_\pi + p'_\pi)_\mu$) remains unchanged, because the vector $(p_\pi + p'_\pi)_\mu$ describes the system as a whole and does not depend on the interaction inside the system. So, when the wave function is varied the l.h.s. remains collinear to the vector $(p_\pi + p'_\pi)_\mu$. In the general case the 4-vector in the r.h.s. changes direction. The equality is valid for an arbitrary choice of wave function only if the vector A^{int}_μ is collinear to the vector $(p_\pi + p'_\pi)_\mu$ in any coordinate system, so that the proportionality factor can be included in the invariant form factor $g(s, Q^2, s')$. This form for A^{int}_μ is unique and most general.

So, we obtain the following integral representation for the pion form factor:

$$F_\pi(Q^2) = \int \mathrm{d}\sqrt{s}\mathrm{d}\sqrt{s'}\varphi(k) g(s, Q^2, s')\varphi(k'). \tag{7}$$

To calculate $g(s, Q^2, s')$ one has to make some physical approximations. We shall perform our calculations in the frame of the relativistic impulse approximation given in terms of invariant form factors. This means that instead of the invariant function $g(s, Q^2, s')$ we shall use the invariant function $g_0(s, Q^2, s')$ which defines the electromagnetic current matrix element for the system of two free particles in the basis (3) with pion quantum numbers:

$$\langle \boldsymbol{P}, \sqrt{s}, J, l, S, m_J | j^0_\mu | \boldsymbol{P}', \sqrt{s'}, J', l', S', m_J{}' \rangle = A_\mu(s, Q^2, s') g_0(s, Q^2, s'). \quad (8)$$

The vector $A_\mu(s, Q^2, s')$ is defined by the current transformation properties (by Lorentz covariance and the current conservation law):

$$A_\mu = (1/Q^2)[(s - s' + Q^2)P_\mu + (s' - s + Q^2)P'_\mu]. \quad (9)$$

In this approach we are dealing with the following integral form of the pion's electromagnetic form factor in the relativistic impulse approximation:

$$F_\pi(Q^2) = \int \mathrm{d}\sqrt{s}\mathrm{d}\sqrt{s'}\varphi(k) g_0(s, Q^2, s')\varphi(k'). \quad (10)$$

Here $g_0(s, Q^2, s')$ is the so-called free two-particle form factor to be derived by the methods of relativistic kinematics [24], $\varphi(k)$ is a phenomenological wave function normalized taking into account the relativistic density of states [24]:

$$\varphi(k) = \sqrt[4]{4(k^2 + M^2)}ku(k), \qquad \int \mathrm{d}k k^2 u^2(k) = 1. \quad (11)$$

Here $u(k)$ is a non-relativistic wave function. The free two-particle form factor in (10) is of the form [24]:

$$\begin{aligned} &g_0(s, Q^2, s') \\ &= \frac{(s + s' + Q^2)Q^2}{2\sqrt{(s - 4M^2)(s' - 4M^2)}} \\ &\quad \times \frac{\theta(s' - s_1) - \theta(s' - s_2)}{\sqrt{1 + Q^2/4M^2}[\lambda(s, -Q^2, s')]^{3/2}} \\ &\quad \times \Big\{ (s + Q^2 + s')[G^u_E(Q^2) + G^{\bar{d}}_E(Q^2)] \cos\omega(s, Q^2, s') \\ &\quad + M^{-1}\xi(s, Q^2, s')[G^u_M(Q^2) + G^{\bar{d}}_M(Q^2)] \\ &\qquad \times \sin\omega(s, Q^2, s') \Big\}, \end{aligned} \quad (12)$$

where

$$\xi(s, Q^2, s') = \sqrt{ss'Q^2 - M^2\lambda(s, -Q^2, s')},$$

$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$, $G^{u,\bar{d}}_E(Q^2)$ and $G^{u,\bar{d}}_M(Q^2)$ are the electric and magnetic single-quark Sachs form factors, θ is the step function, $\omega(s, Q^2, s') = \omega_1 + \omega_2$ is the Wigner rotation parameter, and

$$\omega_1 = \arctan \frac{\xi(s, Q^2, s')}{M[(\sqrt{s} + \sqrt{s'})^2 + Q^2] + \sqrt{ss'}(\sqrt{s} + \sqrt{s'})},$$

$$\omega_2 = \arctan \frac{\alpha(s, s')\xi(s, Q^2, s')}{M(s + s' + Q^2)\alpha(s, s') + \sqrt{ss'}(4M^2 + Q^2)},$$

where $\alpha(s, s') = 2M + s^{1/2} + s'^{1/2}$, and $s_{1,2}$ are given by the kinematic constraint [7,13]

$$\begin{aligned} s_{1,2} &= 2M^2 + \frac{1}{2M^2}(2M^2 + Q^2)(s - 2M^2) \\ &\quad \mp \frac{1}{2M^2}\sqrt{Q^2(Q^2 + 4M^2)s(s - 4M^2)}. \end{aligned}$$

While obtaining (7) and (10) we did not use a fixed coordinate frame (for example, a Breit frame) or fixed ("good") current components, as one usually does in other RHD approaches [18]. In this respect, our calculations are Lorentz covariant. Our current matrix element satisfies conservation laws, so that the current operator of composite system does contain the contribution not only of one-particle currents but of two-particle currents too [24] (in its covariant part A^{int}_μ in (6)).

We are proceeding based on the physical assumption that CQM as a phenomenological model has to describe correctly: (a) the charge conservation law $F_\pi(0) = 1$; (b) the pion MSR $\langle r^2_\pi \rangle^{1/2}$, which is measured in a model independent way; (c) the lepton decay constant f_π; (d) the non-relativistic limit; (e) the chromodynamical asymptotics at $Q^2 \to \infty$. All these conditions are fulfilled in our approach.

The functional form of the quark form factor is motivated by the asymptotic condition (details can be found in [27]). By analogy with [10] and with the scaling of the nucleon form factors we write

$$G^q_E(Q^2) = e_q f(Q^2), \quad G^q_M(Q^2) = (e_q + \kappa_q) f(Q^2), \quad (13)$$

where e_q is the quark charge and κ_q is the quark anomalous magnetic moment. However, we do not use for $f_q(Q^2)$ the form of [10] but that of [11]:

$$f_q(Q^2) = \frac{1}{1 + \ln(1 + \langle r^2_q \rangle Q^2/6)}. \quad (14)$$

Here $\langle r^2_q \rangle$ is the quark MSR. Let us briefly discuss the motivation for choosing the explicit form (14). One of the features of our approach is the fact that the form-factor asymptotic behavior at $Q^2 \to \infty$, $M \to 0$ does not depend on the choice of the wave function in (10) and is defined by the relativistic kinematics of two-quark system only [12]. In the point-like quark approximation (κ_q=0, $\langle r^2_q \rangle$= 0) the asymptotics coincides with that described by quark counting laws [28]: $F_\pi(Q^2) \sim Q^{-2}$. The form (14) gives logarithmic corrections to the power-law asymptotics obtained in QCD. So in our approach the form (14) for the quark form factor gives the same asymptotics as in QCD. Let us notice, however, that the main results of the present paper do not depend on the actual form of the quark form factor.

As the quark interaction potential is not known from first principles, CQM usually deals with model potentials and wave functions depending on fitting parameters. To calculate the pion form factor we use the following wave functions for the ground state of the quark–antiquark system.

(1) The harmonic oscillator (HO) wave function (see e.g. [3]):

$$u(k) = N_{\rm HO}\exp\left(-k^2/2b^2\right). \quad (15)$$

(2) The power-law (PL) wave function (see e.g., [9]):

$$u(k) = N_{\rm PL}(k^2/b^2+1)^{-3}. \quad (16)$$

(3) The wave function with linear confinement and Coulomb-like behavior at small distances [29]:

$$u(r) = N_T{\rm e}^{-\alpha r^{3/2}-\beta r}, \quad \alpha = \frac{2}{3}\sqrt{Ma}, \quad \beta = \frac{Mb}{2}. \quad (17)$$

In (17), a and b are the parameters of the linear and Coulomb parts of the potential respectively. $b=(4/3)\alpha_{\rm s}$, $\alpha_{\rm s} = 0.59$ on the scale of the light mesons mass.

One can see from (10)–(17) that we use the standard CQM parameters: the constituent-quark masses $M_u = M_d = M$, the u- and $\bar{d}$-quark anomalous magnetic moments $\kappa_u, \kappa_{\bar{d}}$ (which enter our equations through the sum $s_q = \kappa_u + \kappa_{\bar{d}}$), the constituent-quark MSR $\langle r_u^2\rangle = \langle r_d^2\rangle = \langle r_q^2\rangle$ and the wave functions parameters b in the models of (15) and (16), and a and b in the model of (17). Let us remark that the relativistic effects of spin rotation are responsible for the contribution of the quark magnetic moments to the charge pion form factor [27].

Let us notice that the electroweak properties of mesons have been discussed by different authors in the framework of CQM in the point-quark approximation ($\langle r_q^2\rangle = 0$, $\kappa_q = 0$) and a consistent description of some processes has been obtained [3,4,6,9,13]. However, there are strong arguments against this approximation. The model independent Gerasimov sum rules [20] indicate the existence of anomalous magnetic moments of the constituent quarks. The anomalous magnetic moments of the quarks appear in the calculations of [2,30]. Let us refer also to our paper [13] where the pion form factor was calculated in a point-like quark model. In [13] our aim was to describe only the electromagnetic properties of the pion, and this was possible with point-like quarks. In that case, we obtained a strong dependence of the form factor on the explicit form of the wave function. However, a simultaneous description of electromagnetic (MSR) and weak (lepton decay constant f_π) properties is impossible in the point-like quark model with sufficient accuracy at realistic values of the parameters. So, in our approach the quark internal structure appears in a natural way. In the case considered in the present paper the form-factor dependence on the wave function is found to be weak.

The parameters in our calculations are of two types. The first type of parameters enter the electromagnetic or weak current of the constituent quark: M, s_q, $\langle r_q^2\rangle$. The second type of parameters characterize the quark interaction (wave functions), b, a. The first parameter type is to be fixed independently of the choice of the model interaction. In other words, the calculation of composite quark systems is analogous to that of composite nuclear systems, e.g., the deuteron. In the calculation of the deuteron electromagnetic properties one fixes the parameters in nucleon form factors independently of the choice of the nucleon–nucleon interaction potential.

Let us now fix the parameters. At present, there are two pion characteristics that can be extracted from the data in a model independent way and with sufficient accuracy: the mean square radius $\langle r_\pi^2\rangle_{\rm exp}^{1/2} = 0.657\pm0.012\,{\rm fm}$ [31], and the lepton decay constant $f_{\pi\rm exp} = 0.1317\pm0.0002\,{\rm GeV}$ [32]. We assume that the calculations for any quark interaction model satisfy (in addition to the description of the particle spectrum) the conditions

$$\langle r_\pi^2\rangle^{1/2} = \langle r_\pi^2\rangle_{\rm exp}^{1/2}, \quad (18)$$

$$f_\pi = f_{\pi\rm exp}. \quad (19)$$

We have used the following forms for the pion MSR and the lepton decay constant [13]:

$$\langle r_\pi^2\rangle = -6\,\frac{{\rm d}F_\pi(Q^2)}{{\rm d}Q^2}\bigg|_{Q^2=0} = \langle r_{\rm r.m.}^2\rangle + \langle r_q^2\rangle, \quad (20)$$

$$f_\pi = \frac{M\sqrt{3}}{\pi}\int\frac{k^2{\rm d}k}{(k^2+M^2)^{3/4}}u(k). \quad (21)$$

In (20) $\langle r_{\rm r.m.}^2\rangle$ is the contribution of the quarks' relative motion and depends on M, s_q and on the wave function parameters; $\langle r_q^2\rangle$ is the part of the pion MSR due to the MSR of the quarks. The lepton decay constant is defined by the wave function parameters and by the mass of the constituent quark.

The details of the derivation of (21) in the framework of the instant form RHD are given in [11]. It is worth to notice that (21) coincides with the expression obtained in the framework of another RHD form, namely light front dynamics in [6]. The factor $(k^2+M^2)^{3/4}$ in the integrand appears as a consequence of the relativistic approach to f_π. In the non-relativistic limit (21) gives the lepton decay constant in terms of the value of the wave function at the origin in the coordinate representation.

The choice of the values of (18) and (19) to fix the parameters has the following reason. As one can see from (20), the mean square radius of the pion is determined by the form-factor behavior near zero. This means that the condition (18) gives, in fact, a constraint for the pion form factor at small Q^2 values. Analogously the constant f_π is connected with the pion form-factor behavior at large momentum transfer. So the conditions (18) and (19) constrain, in fact, the pion form-factor behavior at small and large momentum transfer.

Thus, the constituent-quark parameters M, s_q and $\langle r_q^2\rangle$ are the same for all the wave functions in (15), (16) and (17). We shall use the relation $\langle r_q^2\rangle \simeq 0.3/M^2$ between the MSR and the mass of the constituent quark [10,16].

Let us consider now the parameter s_q and the parameters of the wave functions. To fix these parameters in the framework of the model under consideration one can use the conditions (18) and (19). A difficulty is the fact that the parameter s_q of the internal quark structure has to be the same for all models in (15), (16) and (17). Because of this we choose s_q and the wave functions parameters in

Table 1. The values of the model parameters for the higher ($M = 0.22\,\mathrm{GeV}$), medium ($M = 0.25\,\mathrm{GeV}$) and lower ($M = 0.33\,\mathrm{GeV}$) groups of curves in Fig. 1. The parameter b in (15) and (16) is in GeV, the parameter a in (17) is in GeV^2. The wave function parameters b and a and the sum s_q of the quark anomalous magnetic moments are derived from the fitting of the pion MSR $\langle r_\pi^2\rangle^{1/2}$=0.657±0.012 fm [31] and the best possible posterior fitting of the value $f_{\pi\mathrm{exp}} = 131.7 \pm 0.2\,\mathrm{MeV}$ [32]

	$M = 0.22$		$M = 0.25$		$M = 0.33$	
	$s_q = 0.0268$		$s_q = -0.0023$		$s_q = -0.1965$	
Model	b, a	f_π	b, a	f_π	b, a	f_π
(15)	0.3500	127.4	0.3069	127.8	0.2558	125.1
(16)	0.6131	131.7	0.5401	131.7	0.4901	131.7
(17)	0.1331	131.7	0.0670	132.1	0.0187	131.7

such a way as to satisfy the condition (18) precisely for all the models (up to experimental errors) and the condition (19) approximately, but with minimal deviation for each model.

The corresponding values are given in Table 1. In such a way we fix all parameters but one: the constituent-quark mass M remains as a fitting parameter. When one changes M the other parameters are changed following the indicated prescription, $\langle r_q^2\rangle$, s_q, a and b being functions of M.

The results of our calculation of the pion form factor using the parameters from Table 1 indicate that the form-factor dependence on the quark interaction model is weak, while the dependence on the constituent-quark mass is rather significant. Our results are presented in Fig. 1. The curves calculated with different wave functions but one and the same quark mass form groups[1]. In Fig. 1 the position of the group changes essentially with the quark mass.

The great accuracy of planned JLab experiments will make it possible to fix the position of "the group" rather accurately and, so, to determine the constituent-quark mass[2]. This estimate (almost) will not depend on the interaction model for quarks in pion.

It is worth to emphasize that the function $f_q(Q^2)$, see (14), enters $F_\pi(Q^2)$ as a multiplier, so the choice of $f_q(Q^2)$ does not influence the relative position of the curves for different M and different model wave functions.

It is possible that the slope of the experimental curve will turn out to be greater than in our groups, so that for different Q^2 the points will belong to different groups.

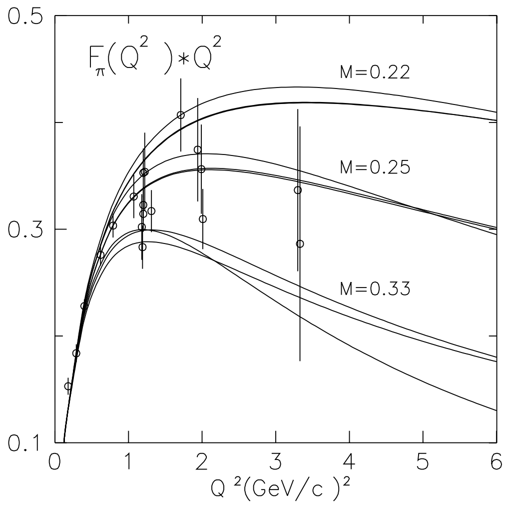

Fig. 1. The π-meson form factor in the range of the JLab experiments. The results of the calculations are for different interaction models and for $M = 0.22, 0.25, 0.33\,\mathrm{GeV}$. The curves with the same mass form a group. The position of a group is defined by the constituent-quark mass

This case will indicate that the constituent-quark mass depends on the momentum transfer, in the spirit of [8].

So our approach gives the possibility to estimate the constituent-quark mass from the experimental pion form factor. Moreover, if the mass has been determined we can estimate the anomalous magnetic moments of the u- and d-quarks using the parameter $s_q = \kappa_u + \kappa_{\bar d}$. To perform this estimate one can use the model independent Gerasimov sum rule [20]: $(e_u + \kappa_u)/(e_d + \kappa_d) = -1.77$. For example, for $M \simeq 0.25\,\mathrm{GeV}$ we obtain $\kappa_u = -0.0285, \kappa_d = -0.0262$; these values are of the order of the values of [20]. The variation of M gives different values of s_q, and thus, of κ_u and κ_d.

The analogous program can be carried out for the kaon.

To conclude, the calculation of the pion form factor in the framework of our approach based on the instant form RHD gives a weak dependence on the interaction model for quarks in the pion, while the dependence on the constituent-quark mass is strong. One can imagine that any approach to the calculation of the form factor with any wave function will give a result close to our result, if the MSR and the lepton decay constant are described well enough. Our results provide a possibility to estimate the parameters of the constituent quarks from JLab experiments.

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[1] A similar result was obtained in the framework of light front dynamics in [9] and the instant form in [11], where it was found that the charge form factor is approximately insensitive for a large class of wave functions. In [9] and [11] the dependence on the constituent-quark mass was not investigated

[2] Let us note that the use of our approach to the existing experimental data (see Fig. 1) gives only a rough estimate for the mass: $0.22 < M < 0.33\,\mathrm{GeV}$. A glance at Fig. 1 gives $M \simeq 0.25\,\mathrm{GeV}$ for the existing experimental data, close to the estimate $M = 0.24\,\mathrm{GeV}$ given in [33]. The first measurements of F_π in JLab [34] give $M \simeq 0.21\,\mathrm{GeV}$ [35].

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